# Automatic generation virtual models of fourbars for visualized e-courses of interactive engineering mechanics 

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#### Abstract

This paper presents an algorithm for automatic generation of cinematically possible virtual models of fourbars for interactive e-courses of engineering mechanics. The author of present paper supposes that the details of this algorithm on the worksheet of the Computer Package Mathcad are hidden from elearner. Generated virtual model of a fourbar and the dimensions of its links appear immediately at the certain region on the worksheet of the Mathcad after fixing the range number and the characteristic angle of a fourbar. The author of this paper gives also detailed instructions to an e-learner for animation the motion of the virtual model of a fourbar on the worksheet of the Mathcad.


Keywords - linkage; e-learning; computer; generation; fourbar; mathematics;

## 1. Introduction

Most of engineering students have the figurative type of thinking that needs to visualize the engineering subject in order to understand it. That is why the application of virtual reality methods to the teaching process of engineering subjects and to study and understand the working processes of various tools of machinery is important. Nowadays there are several environments of computer programs (Mathematica, Matlab, Scilab, Mathcad, etc), that can be used for this purpose.

Choosing the program for application of virtual reality methods depends on the skills teacher, on the level of e-learners and on the user friendliness of the program environment. Department of Engineering in Estonian University of Life Sciences has decided to realize the virtual reality methods in the environment of the Computer Package Mathcad by the following reasons: 1. book style writing of automatically computing formulae is convenient for students, 2. simple procedure for animation of figures and formulas, 3. possibility for experimentation with lecture materials by changing the given values of parameters with observation the results of computation. 4. possibility for animation the motion of simple virtual objects, 5 . possibility for use the graphics from another Windows application, 6. satisfactory text editor, 7. possibility to associate Mathcad with the e-learning environments.

Papers [1, 2] give a review of the experiences on visualization of e-courses of mathematics and on the virtual reality technology based studying the working processes elements of agricultural machinery. Papers [3, 5, 6] deal with the virtual reality technology based synthesis a manipulator, with different prescribed motion, for press manure removals. Paper [4] describes a method of virtual reality for creating a disk-ridging tool for wide row potato field tillage machine.

Teachers of the e-courses of engineering mechanics have to compose large number of cinematically possible linkages for their interactive analysis by e-learners. This paper presents the algorithm for automatic generation of cinematically possible virtual models of fourbars. Generated virtual fourbars may be directly offered for interactive analysis by e-learners or used for composing fourbar based complex linkages for this analysis.

## 2. Algorithm for generation of fourbars on the worksheet

## of the Computer Package Mathcad

### 2.1. Determination the positions of the moving pivots

Let us consider a fourbar in Fig. 1, where the pivots O and C are immovable and the pivots A, E and D are moving. At that the pivot D is rigidly clamped to the connecting rod AE and can be used for joining to it another link or linkage. If crank OA can rotate around immovable pivot O , then the dimensions of a linkage are cinematically possible. The position of the crank OA determines the crank angle $\varphi$. The pivot D draws on the xy-plane a connecting rod curve.


Figure 1. A fourbar
Let us suppose that the range number of the fourbar is No and the value of the angle between links AE and AD is $\Theta$ (Fig. 1) and define the generation parameters $\mathrm{k}=1,2,,, 5 ; \mathrm{s}=1,2, \ldots, 4$,

$$
u_{k}=0.1+\frac{2}{3}\left|\cos \left(\frac{k+10 N o}{}\right)+\frac{1}{3} \cos \left(\frac{k+30 N o}{3}\right)+\frac{1}{5} \cos \left(\frac{k+50 N o}{5}\right)\right| \quad, v_{\mathrm{s}}=u_{\mathrm{s}} .
$$

It is known [7] that the satisfaction of Grashof condition $S+L \leq P+Q$, where $S$ is the shortest length, $L$ - the longest length of the links and $\mathrm{P}, \mathrm{Q}$ - the lengths of the rest two links, guarantees the of cinematic possibility of a linkage. Let the vector $v=\left(v_{1}, v_{2}, v_{3}, v_{4}\right)$ has after sorting its elements in ascending order the elements ( $v_{1}^{\prime}, v_{2}^{\prime}, v_{3}^{\prime}, v_{4}^{\prime}$ ). To make possible the satisfaction of the Grashof condition, let us define the following dimensions for a fourbar (Fig. 1): $\mathrm{OA}=v_{1}^{\prime}, \mathrm{AE}=v_{4}^{\prime}, \mathrm{EC}=v_{2}^{\prime}, \mathrm{OC}=v_{3}^{\prime}$, where $\mathrm{OA}, \mathrm{AE}, \mathrm{EC}$, OC are the lengths of the corresponding links $\mathrm{OA}, \mathrm{AE}, \mathrm{EC}, \mathrm{OC}$. Let us additionally define $\mathrm{AD}=u_{5}$, where AD is the length of the link AD . Co-ordinates of the pivot A are $x_{\mathrm{A}}(\varphi)=\cos \varphi$ and $y_{\mathrm{A}}(\varphi)=\sin \varphi$. The co-ordinates of the pivot E are satisfy the equations of restrictions

$$
\begin{equation*}
F_{1}\left(x_{\mathrm{E}}(\varphi), y_{\mathrm{E}}(\varphi), \varphi\right)=0, \quad F_{2}\left(x_{\mathrm{E}}(\varphi), y_{\mathrm{E}}(\varphi), \varphi\right)=0, \tag{1}
\end{equation*}
$$

where

$$
\begin{gathered}
F_{1}\left(x_{\mathrm{E}}(\varphi), y_{\mathrm{E}}(\varphi), \varphi\right)=\left[x_{\mathrm{E}}(\varphi)-x_{\mathrm{A}}(\varphi)\right]^{2}+\left[y_{\mathrm{E}}(\varphi)-y_{\mathrm{A}}(\varphi)\right]^{2}-A E^{2}, \\
F_{2}\left(x_{\mathrm{E}}(\varphi), y_{\mathrm{E}}(\varphi), \varphi\right)=\left[O C-x_{\mathrm{E}}(\varphi)\right]^{2}+\left[y_{\mathrm{E}}(\varphi)\right]^{2}-E C^{2} .
\end{gathered}
$$

The system of equations (1) can be solved by Mathcad solve block. For determination the co-ordinates of the pivot D the Mathcad program [7]

$$
V(x, y, \alpha, p)=\left\{\begin{array}{l}
v_{O} \leftarrow\left(\begin{array}{cccccc}
x & x & x & x & x & x \\
y & y & y & y & y & y
\end{array}\right)^{T}  \tag{2}\\
I \leftarrow\left(\begin{array}{cccccc}
0 & 0.95 & 0.85 & 1 & 0.85 & 0.95 \\
0 & 0 & 0.04 & 0 & -0.04 & 0
\end{array}\right)^{T} \\
\Omega \leftarrow\left(\begin{array}{ccc}
\cos (\alpha) & \sin (\alpha) \\
-\sin (\alpha) & \cos (\alpha)
\end{array}\right) \\
v_{O}+p \cdot I \cdot \Omega
\end{array}\right.
$$

for drawing geometric vector's on the Mathcad worksheet can be used. Here x and y are the co-ordinates of the beginning of the geometric vector, $\alpha$ - angle of its direction and $p$ - the module of this vector. The function $V(x, y, \alpha, p)$ returns on the Mathcad worksheet a matrix, with two columns $V(x, y, \quad, p)^{\langle 1\rangle}$ and $V(x, y,, p)^{\langle 2\rangle}$ (abstract vectors). On the Mathcad worksheet the angle of direction of the geometric vector AE (Fig. 1) is determined by formula

$$
\alpha(\varphi)=\operatorname{angle}\left[x_{\mathrm{E}}(\varphi)-x_{\mathrm{A}}(\varphi), y_{\mathrm{E}}(\varphi)-y_{\mathrm{A}}(\varphi)\right]
$$

Let us denote

$$
F_{\mathrm{E}}(\varphi)=V\left(x_{\mathrm{A}}(\varphi), y_{\mathrm{A}}(\varphi), \alpha(\varphi), A E\right), F_{\mathrm{D}}(\varphi)=V\left(x_{\mathrm{A}}(\varphi), y_{\mathrm{A}}(\varphi), \alpha(\varphi)+\Theta, A D\right)
$$

Abstract vector's $F_{E}()^{\langle 1\rangle}, F_{E}()^{\langle 2\rangle}$ and $F_{D}()^{\langle 1\rangle}, F_{D}()^{\langle 2\rangle}$ image on the Mathcad worksheet the geometric vector's AE and AD , corresponding to the links AE and AD in Fig. 1, accordingly. Fig. 2 shows these geometric vectors, when $\varphi=60^{\circ}, \Theta=25^{\circ}, \mathrm{AE}=0.7772 \mathrm{~m}, \mathrm{AD}=0.8674 \mathrm{~m}$.


Figure 2. Geometric vectors AE and AD , corresponding to the links AE and AD
It follows from program (2) that the fourth elements of the abstract vectors $F()^{\langle 1\rangle}$ and $F()^{\langle 2\rangle}$ represent the coordinates of the pivot D i.e.

$$
x_{D}()=F()^{\langle 1\rangle_{4}}, \quad y_{D}()=F()^{\langle 2\rangle_{4}}
$$

### 2.2. Composition of the virtual model of a linkage

Let us define the matrixes

$$
A\left(\quad=\left(\left.\begin{array}{cc}
0 m & 0 m \\
x_{A}() & y_{A}() \\
x_{E}() & y_{E}() \\
O C & 0 m \\
x_{E}() & y_{E}() \\
x_{A}() & y_{A}() \\
0 m & 0 m
\end{array} \right\rvert\,, \quad B()=\left(\begin{array}{ccc}
x_{E}() & y_{E}( & ) \\
x_{D}() & y_{D}( & ) \\
x_{A}() & y_{A}(t)
\end{array}\right)\right.\right.
$$

The columns (abstract vectors) of matrixes $A(\varphi)$ and $B(\varphi)$ draw on the worksheet of the Computer Package Mathcad the virtual model of the fourbar. Fig. 3 shows this model, when $\mathrm{OA}=0.1938 \mathrm{~m}, \mathrm{AE}=$ $0.7772 \mathrm{~m}, \mathrm{EC}=0.4257 \mathrm{~m}, \mathrm{OC}=0.6259 \mathrm{~m}, \mathrm{AD}=0.8674 \mathrm{~m}, \varphi=60^{\circ}, \Theta=25^{\circ}$. On the worksheet of the Mathcad motion of the fourbar can be easily animated.


Figure 3. Virtual model of a fourbar with connection rod curve

## 3. Instructions for e-learners

A teacher has to upload the file with the program of generation of fourbars into the server of the e-learning environment and make special icon in the e-course for downloading this file. Mathcad worksheet allows hiding the algorithm of generation of fourbars. If a e-learner has the Computer Package Mathcad, installed in his computer, then Mathcad opens the downloaded file automatically and an e-learner can see only certain part of the Mathcad worksheet, shown in Fig. 4. If to change the value of the range number of a fourbar No and (or) the angle $\Theta$ on the Mathcad worksheet (Fig. 4) then the hidden program generates immediately a new fourbar and connecting rod curve. If for some values of No the generated fourbar is not cinematically possible, then the the comment "The linkage is not cinematically possible" appears. In the case of abnormal motion of a fourbar one can change the value of No or the initial values of $x_{\mathrm{E}}$ and $y_{\mathrm{E}}$, from which the Mathcad solve block begins to specify the values of these coordinates.

Range number of a fourbar: $\mathrm{No} \equiv 110$.
Value of the angle: $\Theta \equiv 25$ (in degrees).

This fourbar has the following dimensions:
$\mathrm{OA}=0.1938 \mathrm{~m}, \mathrm{AE}=0.7772 \mathrm{~m}$,
$\mathrm{EC}=0.4257 \mathrm{~m}, \mathrm{OC}=0.6259 \mathrm{~m}$,
$\mathrm{AD}=0.8674 \mathrm{~m}$


Comment $=$ "FOURBAR IS CINEMATICALLY POSSIBLE"


Figure 4. A part of worksheet of the Mathcad with generated virtual model of a fourbar
It is possible to animate the motion of the generated virtual model of a fourbar on the Mathcad worksheet. For that purpose one have to open the Mathcad window "Animate" (Fig. 5), choose for FRAME To $=200$ and the virtual image of fourbar on the worksheet of Mathcad and click the button "Animate". The process of animation begins. At the end of animation the video clip with the animation appears. This video clip may be replayed or saved into a computer as a file with extension "avi".


Figure 5. Mathcad's window "Animate"

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