# USING THE LAGRANGE EQUATIONS IN THE BRAKING DYNAMICS OF SYSTEM TRACTOR-TRAILER

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***Abstract***: *The paper elaborates a mathematical model to calculate the braking parameters of tractor-trailer system by using Lagrange equations. This mathematical model can be used to graphically trace the evolution in time of the following parameters: braking acceleration, speed during braking and distance covered by tractor during braking. There are presented applications of this method for more values of the adhesion coefficient and for two values of the initial braking speed for a tractor with a power of 52 kW.*
***Key words:*** *tractor, tractor-trailer system, braking, braking performance.*

## ESTABLISHING THE MATHEMATICAL MODEL FOR THE CALCULATION OF BRAKING PARAMETERS

The study of the tractor-trailer brake system calls for solving two dynamics problems. The tractor-trailer system stands for a mechanical system with a finite number of degrees of freedom. That is why the study related to the braking dynamics of the tractor-trailer system uses the general principles regarding the dynamics of mechanical systems. The complexity of the study consists of accurately evaluating the degrees of freedom and of determining the forces acting upon the system, i.e. of establishing the calculation diagram.

As resulting from *Tractor's Theory*, the forces acting upon the tractor can not exceed certain values resulted from the adhesion condition of the rolling system to the road. For the calculation of the maximum external forces from the adhesion condition the vertical reactions at the tractor's wheels must be calculated.

The rectilinear tractor motion is analyzed under the conditions of intense braking. The calculation diagram of tractor-trailer brake system is presented in figure 1.

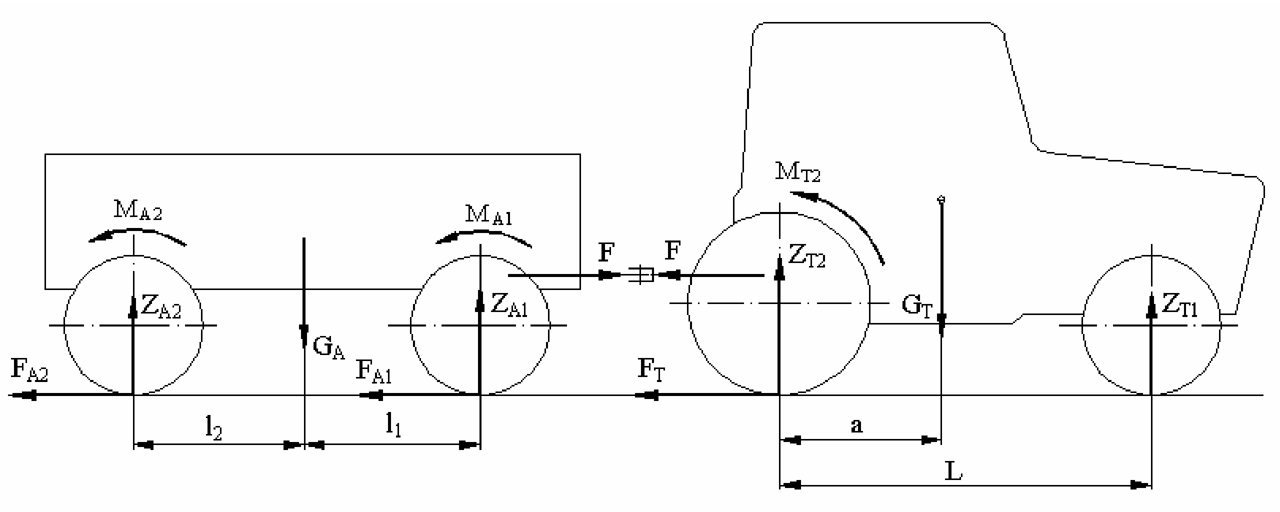


Fig. 1. *Forces and moments acting on a tractor-trailer system during braking.*

In the general case, the following forces act upon the tractor-trailer system, which leads to speed reduction: braking, rolling resistance, aerodynamic resistance, frictions in the bearings of the running system etc. The value of braking forces, under the conditions of a braking of maximum intensity, represents 98% of the total number of forces slowing down the speed. That is why the braking forces alone will be subsequently considered. It is admitted that wheels slipping is very low. In this case, at any moment, the following condition is met:

$$\omega = v/r\,, \tag{1}$$

where: $\omega$ is the wheels angular speed; $v$ – translational velocities of the wheel's center; $r$ – wheel radius.

Lagrange equation of second order is used to elaborate the tractor-trailer system's equation of motion. It is admitted that coupling between tractor and trailer is rigid and clearance-free. The system analyzed has a single degree of freedom, determined by the generalized coordinate x.

The kinetic energy of the system is calculated with the relation:

$$E = \frac{m_T \dot{x}^2}{2} + \frac{m_A \dot{x}^2}{2} + \frac{\sum J_{Ti} \dot{\theta}_{Ti}^2}{2} + \frac{\sum J_{Aj} \dot{\theta}_{Tj}^2}{2},$$

where:$m_T$ denotes the tractor's mass; $m_A$ – the trailer's (attachment's) mass; $J_{Ti}$ – moment of inertia of the tractor's wheel; $J_{Aj}$ – moment of inertia of the trailer's wheel.

Taking into account the relation (1), $\dot{\theta} = \dot{x} / r$ , and

$$E = \frac{\left(m_{T.\ red} + m_{A.\ red}\right)\dot{x}^2}{2}, \tag{2}$$

where: $m_{T.\ red} = m_T + \frac{\sum J_{Ti}}{r_i^2}$ – tractor's reduced mass;

$m_{A.\ red} = m_A + \frac{\sum J_{Aj}}{r_j^2}$ – trailer's reduced mass.

The generalized force is determined with the virtual mechanical work. Provided the generalized coordinate increases with $\delta x$, the wheel rotates with angle $\delta\theta = \delta x / r$ .

The virtual mechanical work of all active forces is calculated for a wheel:

$$\delta L = -M \cdot \delta\theta = -\frac{M}{r}\delta x. \tag{3}$$

In the last relation the braking force $F$ is not included as this force is applied in the instantaneous rotation center of velocities and, subsequently, the mechanical work is zero.

The generalized force of the tractor-trailer system is determined with the following relation:

$$Q = \frac{\delta L}{\partial x} = -\sum \frac{M_i}{r_i}. \tag{4}$$

The equation of motion of the tractor-trailer system is subsequently deduced, using Lagrange equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{x}}\right) - \frac{\partial E}{\partial x} = Q.$$

Using the relation (2) there is obtained:

$$\frac{\partial E}{\partial \dot{x}} = \left(m_{T.red} + m_{A.red}\right)\dot{x}; \quad \frac{\partial E}{\partial x} = 0.$$

Consequently the equation will be:

$$\ddot{x}\left(m_{T.\ red} + m_{A.\ red}\right) = -\sum \frac{M_i}{r_i}. \tag{5}$$

By solving the equation (5) as related to $\ddot{x}$, the relation for the braking deceleration of the tractor-trailer system is obtained:

$$\ddot{x} = -\frac{1}{m_{T.red} + m_{A.red}} \sum \frac{M_i}{r_i} \qquad \text{or}$$

$$\ddot{x} = -\frac{1}{m_{T.red} + m_{A.red}} \sum F_i . \tag{6}$$

It is considered that the braking force at tractor's and trailer's wheels, $F_i = M_i/r_i$, varies according to an exponential law:

$$\sum F_i = F(t) = \left(F_{T.max} - fZ_T\right)\left(1 - e^{-\alpha t}\right) + \left(F_{A.max} - fZ_A\right)\left(1 - e^{-\alpha t}\right) + \left(G_T + G_A\right)f. \tag{7}$$

where: $F_{T.\max}$ denotes maximum braking force of the tractor; $F_{A.\max}$ – maximum braking force of the trailer; $Z_T$ – weight distributed on the tractor's braked axle; $Z_R$ – weight distributed on the tractor's braked axle/axles; $G_T$– tractor's total weight; $G_A$ – tractor's total weight; $f$ – rolling resistance coefficient; $\alpha = 5/t_0$– exponent depending on the time $t_0$ of braking complete actuation.

Integrating equation (6) there is obtained the equation of system's speed:

$$\dot{x} = -\frac{1}{m_{T.red} + m_{A.red}} \int_0^t \sum F_i(t)\mathrm{d}t + \dot{x}_0 , \tag{8}$$

where $\dot{x}_0$ is the initial braking speed (speed prior to braking).

Provided the last relation is equaled to zero, the braking time $t_b$ is obtained until a complete stop.

The braking distance is obtained by integrating the equation (8):

$$x_F = \int_0^{t_F} \left[ \frac{1}{m_{T.red} + m_{A.red}} \int_0^{t_F} \sum F_i(t)\mathrm{d}t + \dot{x}_0 \right] \mathrm{d}t. \tag{9}$$

In the light of the bond insulation principle, the tractor-trailer system interaction is analyzed during the braking process. In figure 1 the binding forces are noted with $F$. Balance equations are written separately for both tractor and trailer:

$$\left.\begin{aligned} m_T\ddot{x} &= -F_T - F; \\ m_A\ddot{x} &= -F_A + F, \end{aligned}\right\} \tag{10}$$

where: $F_T$ denotes total braking force of the tractor: $F_T = F_{T1} + F_{T2}$;

$F_A$ – total braking force of the trailer: $F_A = F_{A1} + F_{A2}$.

It is define the concept of partial braking acceleration

$$q_T = \frac{F_T}{m_T};\ q_A = \frac{F_A}{m_A}.$$

There result that the partial braking accelerations represent in fact tractor's acceleration as well as trailer's acceleration in case of independent braking.

Using the last two notations, equations (10) become:

$$\left.\begin{aligned} m_T\ddot{x} &= -m_T q_T - F; \\ m_A\ddot{x} &= -m_A q_A + F. \end{aligned}\right\} \tag{11}$$

By eliminating $\ddot{x}$ from the system (2), the force $F$ from the coupling element is determined:

$$F = \frac{(q_A - q_T)m_T m_A}{m_T + m_A}. \tag{12}$$

From the equation (11) and (12) there results that, when brake intensity for tractor and trailer are equal ($q_T = q_A = \ddot{x}$), the force from the hitch $F = 0$.

If $q_T < q_A$, $F > 0$, on the hitch acts a traction force.

If $q_T > q_A$, $F < 0$, on the hitch acts a pushing force.

As results from relation (12), force $F$ is proportional to the difference of partial accelerations for both tractor and trailer; similarly, the force depends on the mass ratio between tractor and trailer.

## APPLICATIONS OF THE ESTABLISHED MATHEMATICAL MODEL UPON A 4X4 TRACTOR OF 52 KW

There are presented the results regarding the calculation of braking parameters for a

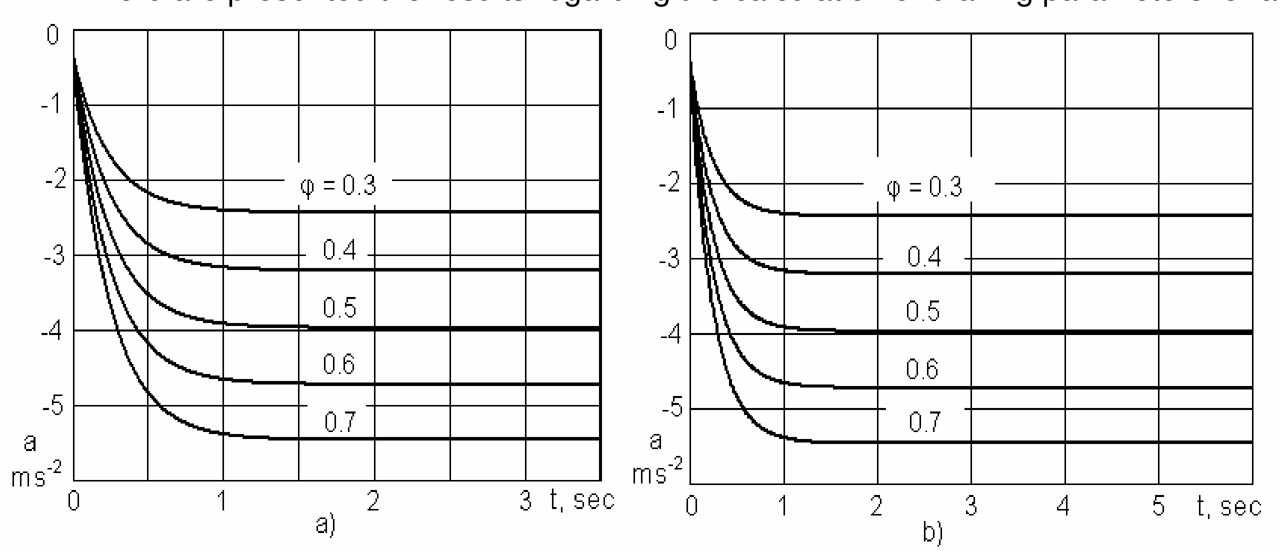


Fig. 2. *Acceleration variation according to time and adhesion coefficient: a) initial braking speed* 27 km/h*; b) initial braking speed* 45 km/h.

U 700 DTC wheeled tractor with a power of 52 kW (70 HP), having the exploitation weight of 47 k N, weight distributed on the rear axle 27.75N and maximum traveling speed of the tractor of 27 km/h. The trailer has the braking system on both axles and the loaded weight of 47 kN. The calculation was done for 5 values of the adhesion coefficient: 0.3, 0.4, 0.5, 0.6, 0.7 and for two values of the initial braking speed: 27 km/h and 45 km/h. The last value of the initial braking speed corresponds to the maximum speed for tractors on public roads admitted by the new Romanian Road Regulations.

The results of the analytical calculation are graphically presented for:

- braking acceleration (fig. 2)
- speed variation during the braking process (fig. 3);
- variation of distance covered during the braking process (fig. 4).

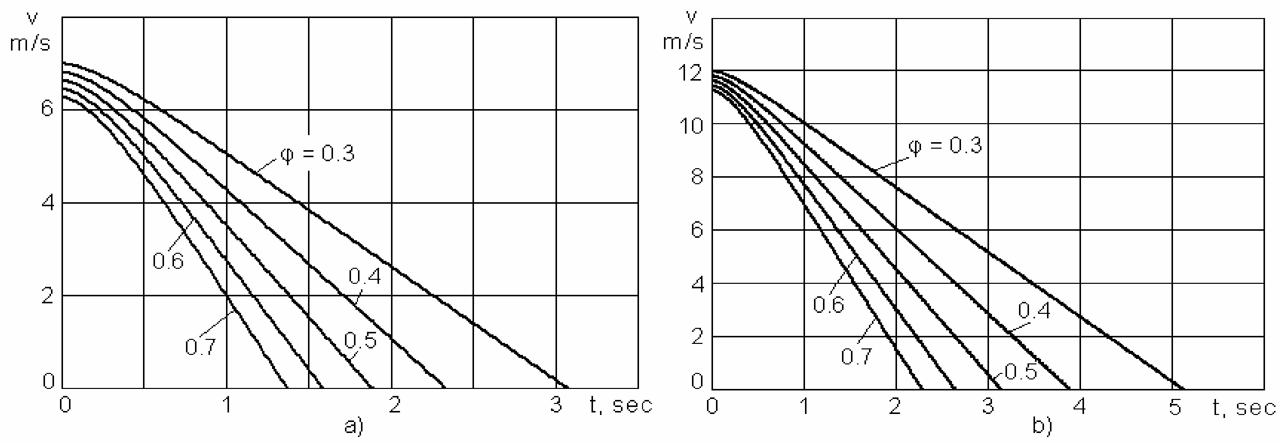


Fig. 3. *Speed variation according to time and adhesion coefficient: a) initial braking speed* 27 km/h*; b) initial braking speed* 45 km/h.

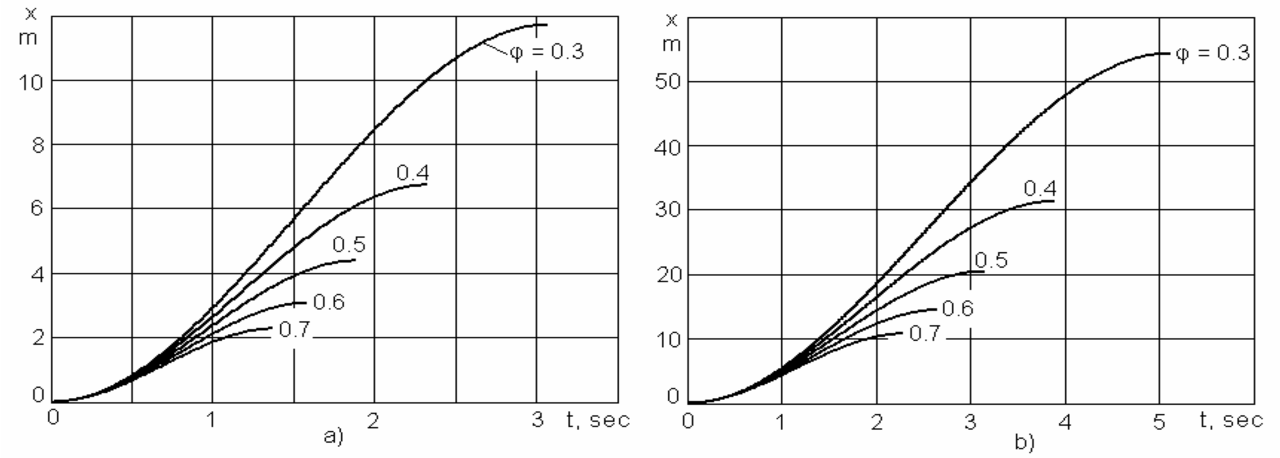


Fig. 4. *Variation of distance covered during braking according to time and adhesion coefficient: a) initial braking speed* 27 km/h*; b) initial braking speed* 45 km/h.

## CONCLUSIONS

The mathematical model established allows the determination of braking parameters for all types of wheeled tractors (4x2 and 4x4) and for various categories of ground, characterized through the adhesion coefficient and through the rolling resistance coefficient.

The braking parameters depend on the initial braking speed an on the adhesion coefficient.

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