AN ANALYSIS OF FRUIT DETACHMENT BY OSCILLATION OF THE STEM AND FRUIT THROUGH VIBRATION

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SUMMARY

A mechanical model of the detachment of fruit, oscillation of the stem and fruit are analyzed in this work, where a more pronounced weight of the fruit, an assumed higher density, and stem with greater rigidity than bond strength were taken.

Detachment of the fruit is caused by tensile force, where it achieves the highest value when the fruit passes through its vertical position, where $\cos \omega t = 1$, ($\cos \varphi = 1$ and that is for $\varphi = 0$).

Detachment force changes with the ratio of branch amplitude and branch (stem) oscillations. It is more difficult to shake off a longer branch (stem) while shaking the fruit rather than those with shorter stems (a problem of centrifugal force).

The shorter the length of the stem, the greater the rigidity and angular frequency; that is, a four times shorter length means two times greater frequency.

It can be concluded that in higher frequencies of the shaker, only the fruit detach, but in lower frequencies the fruit along with the stem detach.

INTRODUCTION

Mechanization of fruit picking is based on the theory of vibration, where every possible form of the oscillation of fruit corresponds to the adequate amplitude and frequency. During mechanical picking those forms of oscillation which achieve the highest force of fruit detachment are chosen.

With oscillation of native trees, the stem can swing at any level, where the fruit stem, in a vertical position of gravity of the fruit, is moved from the vertical to another level. A mathematical analysis of mobilizing the gravity of the fruit in relation with the branch as suspension, along with some limitations leads to errors in the final results. Fruit falling from branches in the front occurs as a result of the effect of inertial forces which occur in shaking at a point of fruit suspension at a horizontal or vertical level. That depends on the expanding force, strength of the connection of the stem to the native branch as well as the characteristics of the stem.

TYPES OF FORMS IN MOVEMENT OF FRUIT

Basic forms in movement of fruit in vibration harvesting are shown in figure 1. According to the theory of vibration, the fruit can be detached through force or momentum. Horizontal force swings the fruit (figure 1. a), whereas vertical causes the fruit to jump (figure 1. c). Detachment by momentum is detachment by bending (figure 1. b) or twisting (figure 1. c). If we observe suspension as a relative relationship of rigidity and strength, that is when rigidity is more pronounced and leads to movement (figures 1. a and 1. c), and more pronounced strength to movement (figures 1. b and 1. d). Conformation leads to complex motion (figure 1. f).

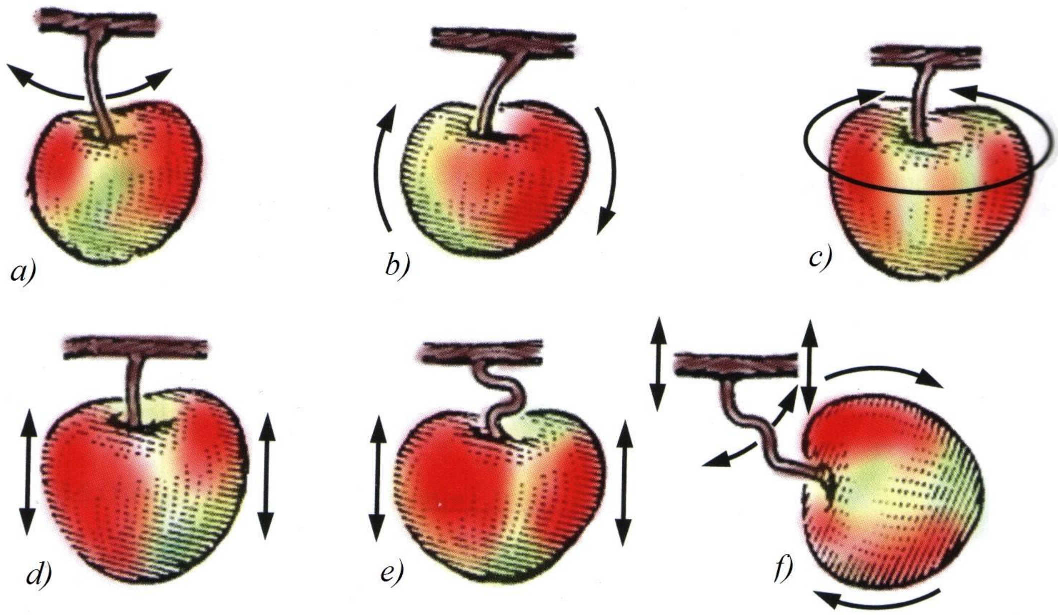

Figure 1. Forms of fruit movement: a) swinging, b) swaying, c) circular, d) vertical jerking(tugging), e) vertical intermittent jerking, f) combined

DETACHMENT OF FRUIT BY OSCILLATION OF THE STEM

Oscillation of the stem in figure 2 can be described as a mathematical oscillator (or a type of pendulum) if the fruit mass is more pronounced than the size (diameter), that is, larger density, but the strength or rigidity of the stem is more pronounced than the steadfastness of the attachment.

With clear oscillation of the fruit, that is, at a fixed point of suspension, the force of acceleration acting on the fruit in a tangential direction has the size: (figure 2.). It is an opposite and equal tangential component of gravity.

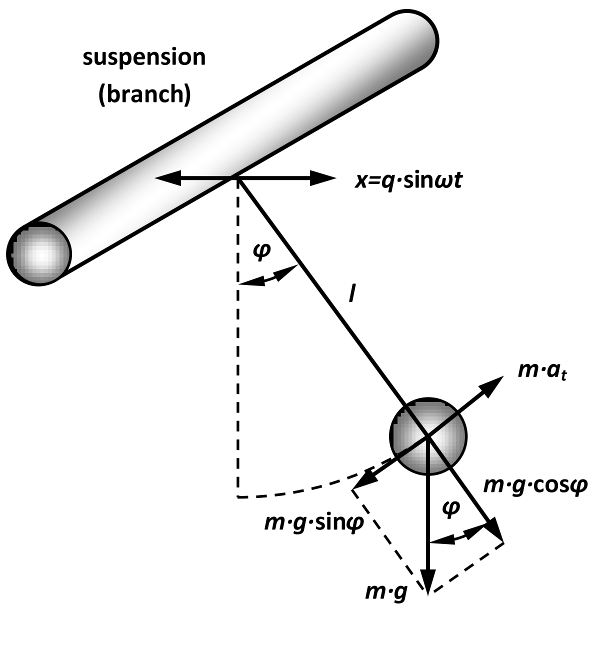

Figure 2. Schema of mathematical model of the oscillator

Application of the second Newton's law in the direction of the tangent is obtained through the equation of motion:

$$m \cdot a_t = -m \cdot g \cdot \sin\varphi \tag{1}$$

Inclusion of the tangential acceleration $a_t = dv/dt$, where is $v = l \cdot \omega$ peripheral speed and $v' = l \cdot \omega'$ its derivation follows:

$$m \cdot l \cdot \omega' = -\, m \cdot g \cdot \sin\varphi \tag{2}$$

Inclusion of $\omega = d\varphi/dt$, that is, $\omega' = \varphi''$ and by dividing with $m \cdot l$ follows the differential equation of the mathematical oscillator:

$$\varphi'' + \frac{g}{l} \cdot \sin\varphi = 0 \tag{3}$$

Given that in this case the mass is not swinging freely, but is influenced harmoniously by the to-and-fro motion of the suspension points in x direction, which is at angle φ with regard to the tangent. Then acceleration x'' affects the mass as well, that is the acceleration force, sized $m \cdot x \cdot \cos\varphi$, then it is also included in expression 2. When shaking the fruit the total tangential forces are balanced.

$$m \cdot l \cdot \varphi'' = -\, m \cdot g \cdot \sin\varphi - m \cdot x'' \cdot \cos\varphi \tag{4}$$

If we take into account that the harmonious activation of point suspension of the pendulum is sinusoidal, the derivations are as follows:

$$x = q \cdot \sin\omega t \tag{5}$$

$$x' = q \cdot \omega \cdot \cos\omega t \tag{6}$$

$$x'' = -\, q \cdot \omega^2 \cdot \sin\omega t \tag{7}$$

where:
q - amplitude point suspension of the fruit on the branch, mm.

By dividing the equation from 4 with $m \cdot l$ and insert the expression 7 we get:

$$\varphi'' + \frac{g}{l} \cdot \sin\varphi = \frac{q}{l} \cdot \omega^2 \cdot \sin\omega t \cdot \cos\omega t \tag{8}$$

In a small deflection angle it can be put at approximately: $\sin\varphi = \varphi$ and $\cos\varphi = 1$, thus the differential equation of oscillations of the fruit takes the form:

$$\varphi'' + \frac{g}{l} \cdot \varphi = \frac{q}{l} \cdot \omega^2 \cdot \sin\omega t \tag{9}$$

Solving the differential equation

The equation of motion in 9, which consists of a free left and forced right part of the oscillation equations can be, as is well-known from the theory of oscillations, dealt with by incorporating $g/l = \omega_n^2$, where ω_n - its own (natural) angular velocity, or through the theory of vibrations of its own angular frequency of fruit with the stem, then we get:

$$\varphi'' + \omega_n^2 \cdot \varphi = \frac{q}{l} \cdot \omega^2 \cdot \sin\omega t \tag{10}$$

The first particular solution which applies to the forced portion of deflection can be found in:

$$\varphi_1 = \varphi_0 \cdot \sin \omega t \tag{11}$$

$$\varphi_1' = \varphi_0 \cdot \omega \cdot \cos \omega t \tag{12}$$

$$\varphi_1'' = -\varphi_0 \cdot \omega^2 \cdot \sin \omega t \tag{13}$$

If we include φ_1' and φ_1'' in the differential equation in 10, by shortening with $\sin \omega t$ we get:

$$\varphi_0 = \frac{q \cdot \omega^2}{l \cdot (\omega_n^2 - \omega^2)} \tag{14}$$

If we insert the obtained φ_0 in expression 10 we then get:

$$\varphi_1 = \frac{q \cdot \omega^2}{l \cdot (\omega_n^2 - \omega^2)} \cdot \sin \omega t \tag{15}$$

We get a full solution to the differential equation when a homogeneous solution of differential equations is added to the forced solution of the deflection, that is, the deflection created by free vibration. A general solution of homogeneous differential equations has the form:

$$\varphi_2 = C_1 \cdot \sin \omega_n t + C_2 \cdot \cos \omega_n t \tag{16}$$

With this we get:

$$\varphi = \varphi_1 + \varphi_2 = \frac{q \cdot \omega^2}{l \cdot (\omega_n^2 - \omega^2)} \cdot \sin \omega t + C_1 \cdot \sin \omega_n t + C_2 \cdot \cos \omega_n t \tag{17}$$

If the constants C_1 and C_2 are determined through initial conditions $t = 0\, t = 0$ and $\varphi = \varphi'' = 0$ we get:

$$C_1 = \frac{\varphi_0 \cdot \omega}{\omega_n};\ C_2 = 0 \tag{18}$$

After inclusion of constants C_1 and C_2 we get a full solution to the equation of the motion of fruit:

$$\varphi = \frac{q \cdot \omega^2}{l \cdot (\omega_n^2 - \omega^2)} \cdot \left(\sin \omega t + \frac{\omega}{\omega_n} \cdot \sin \omega_n t \right) \tag{19}$$

In equations 3 and 9 ratio *g/l* is its own angular frequency ω_n^2 and this is true when the mass of suspension m_p (stems) is negligible in relation to the fruit mass, thus the total stem and fruit mass in the numerator is divided with the fruit mass in the denominator.

$$\omega_n = \sqrt{\frac{g \cdot m_u}{l \cdot m}} = \sqrt{\frac{c}{m}} \tag{20}$$

where:
c - rigidity of the stem, N/m
m_u - total mass, that is, fruit and stem mass, kg
m - fruit mass, kg.

Expression 20 is somewhat more complicated but more general (which is true for the others as well), so it is as such more common in the theory of vibration. In this way, the first expression can be applied for picking plums, but the second for picking cherries.

Its angular frequency during vegetation decreases as the fruit mass of the recently fertilized flower grows, so until full maturity when rigidity of the connection decreases during withering of the stem itself. As we wish to pick healthier, larger and more mature fruits we assume its angular frequency $\omega_n \approx 0$. Furthermore, if oscillation of the fruit is ignored, which as it develops oscillation subsides, then expression 19 is simplified in the following way:

$$\omega_n^2 - \omega^2 \approx 0 \tag{21}$$

We assume that $\omega_n \approx 0$ is then another term in the brackets in expression 19:

$$\frac{\omega}{\omega_n} \cdot \sin \omega_n t \tag{22}$$

However, from the first term the equation of angular deflection in fruit oscillation is as follows:

$$\varphi = -\frac{q}{l} \cdot \sin \omega t \tag{23}$$

By differentiating the equations of motion by time we get the angular velocity and acceleration of fruit:

$$\varphi' = -\frac{q}{l} \cdot \omega \cdot \cos \omega t \tag{24}$$

$$\varphi'' = -\frac{q}{l} \cdot \omega^2 \cdot \sin \omega t \tag{25}$$

As the general expression for the circumferential velocity is $v = r \cdot \omega$ and acceleration $a = r \cdot \varepsilon$ where the values for oscillation is $r = l$, and multiplying the above expressions with the same radius curve trajectory l in meters:

$$v = l \cdot \varphi' = -q \cdot \omega \cdot \cos \omega t \tag{26}$$

$$a = l \cdot \varphi'' = q \cdot \omega^2 \cdot \sin \omega t \tag{27}$$

CALCULATION OF ACCELERATION AND FORCE OSCILLATIONS

The components of inertial forces shaking the fruit, which are directed tangentially and normally for trajectory, can be determined according to figure 3 from the expression:

$$F_n = m \cdot a_n = m \cdot l \cdot \varphi'^2 \quad \text{- normal inertial force} \tag{28}$$

$$F_t = m \cdot a_t = m \cdot l \cdot \varphi'' \quad \text{- tangential inertial force} \tag{29}$$

where:

$a_n = l \cdot \varphi'^2$ - normal acceleration, m/s²

$a_t = l \cdot \varphi''$ - tangential acceleration, m/s².

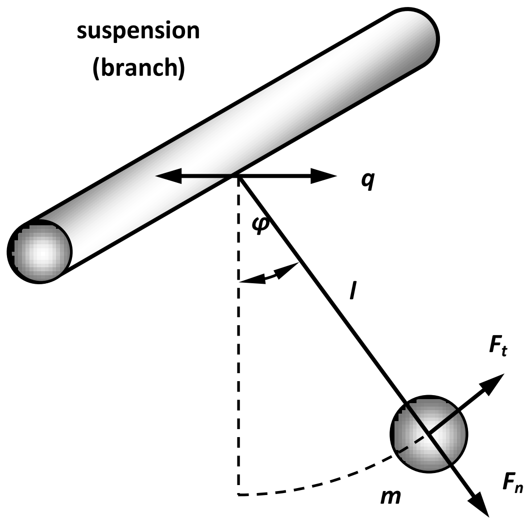

Figure 3. Acceleration force of oscillating fruit

The last two equations included in previous chapters 26 and 27:

$$F_n = m \cdot \frac{q^2}{l} \cdot \omega^2 \cdot \cos^2 \omega t \tag{30}$$

$$F_t = -m \cdot q \cdot \omega^2 \cdot \sin \omega t \tag{31}$$

where:
q - amplitude point suspension stems of the fruit on the branch, mm.

Detachment of the fruit is caused by tensile force F_n which acts parallel to the stem. Vertically acting force F_t does not directly affect detachment. By increasing angular velocity oscillations indirectly increase F_n as centrifugal force. It is like speed, minimal in the end positions, and the largest in the vertical. The same expression can be reached via centrifugal force in rotation:

$$F_n = \frac{m \cdot v^2}{r} \tag{32}$$

where:
v - peripheral rotation speed, m/s
r - radius of circular trajectory, m.

If we include the peripheral rotation speed as $v = r \cdot \omega = l \cdot \omega$ where radius r is equal to the length of the stem l and ω is variable as cosine, this sequence follows:

$$F_n = m \cdot l \cdot \omega^2 \cdot \cos^2 \varphi \tag{33}$$

As this equation is $\varphi = \omega \cdot t$, and the maximum amplitude is $q = l$ the same equation in 30 is as follows:

$$F_n = m \cdot q \cdot \omega^2 \cdot \cos^2 \omega t \tag{34}$$

The force F_n reaches its largest value every time the fruit passes through the vertical position where $\cos \omega t = 1$ ($\cos \varphi = 1$ and that is for $\varphi = 0$), so the equation in 30 is simplified:

$$F_{n\max} = m \cdot \frac{q^2}{l} \cdot \omega^2 \tag{35}$$

From this expression we can discern that in horizontal branch motion the force of detachment is created independently not only from the fruit mass *m* but also its acceleration $q \cdot \omega^2$, as is the case with vertical branch motion.

Apart from that, the force changes with the ratio of *q/l*, that is, the amplitude of the branch at the location where the fruit is attached and the distance of the brunt of the fruit from the branch. As the distance of oscillation *l* is further, that is at a constant weight of the fruit, the frequency of shaking and branch amplitude of the lesser force F_n causes the detachment of fruit. Thus, fruit with longer stems are harder to shake off than fruit with shorter stems (a problem of centrifugal force).

This postulate is confirmed by observing practical experiments during harvest. The condition of detachment is introduced when the tensile force must be:

$$F_{n\max} \geq F \qquad \text{- suspension force (accretion)} \tag{36}$$

equation 35 can also be theoretically calculated, necessary frequency of shaking f in Hz in this way:

$$\omega^2 = \frac{F \cdot l}{m \cdot q^2}\ ;\ \omega = \sqrt{\frac{F \cdot l}{m \cdot q^2}} \tag{37}$$

$$f = \frac{\omega}{2 \cdot \pi} = \frac{1}{2 \cdot \pi \cdot q} \cdot \sqrt{\frac{F \cdot l}{m}} \tag{38}$$

DETACHMENT OF FRUIT BY OSCILLATION

With fruit maturation the attachment of the stem to the fruit is weakened, and the stem gains rigidity by drying. This leads to oscillation of the fruit only more often.

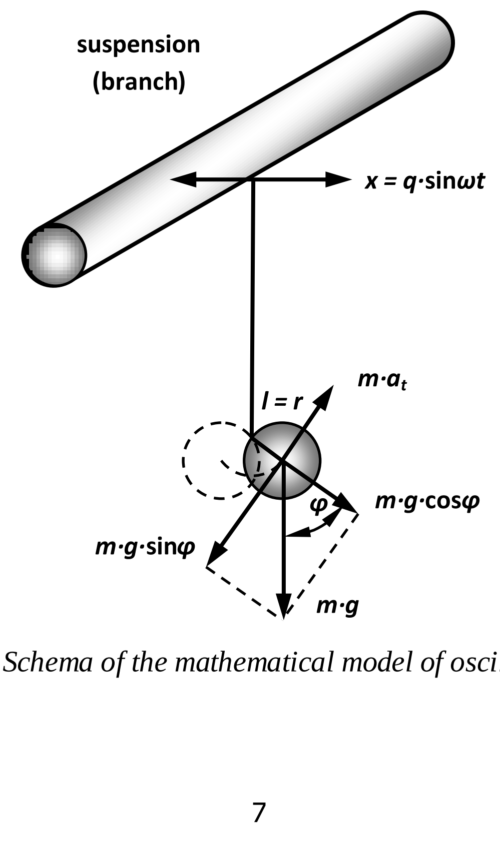

Figure 4. Schema of the mathematical model of oscillating fruit

At the first described oscillation the branch swing *l* (in metres) is equal to the sum of the length of the stem and radius of the fruit, here is a branch with an equal radius $l = r$, so equation 20 along with a negligible mass of the stem takes on the form:

$$\omega_n = \sqrt{\frac{c}{m}} = \sqrt{\frac{g \cdot m_u}{l \cdot m}} = \sqrt{\frac{g}{r}} \tag{39}$$

Multiplying length reduction leads to an increase in rigidity *c* as well as in its own angular frequency of free oscillations ω. Therefore for instance a four times less length $r = l/4$ has a twice as large frequency.

The shakers in which we increase the angular frequency can also be decreased for achieving the necessary detachment force by the same amount (according to 35), while the influence of reducing length *l* is less significant (root).

$$F_{n\max} = m \cdot \frac{q^2}{l} \cdot \omega^2 \tag{40}$$

From all of the above mentioned we can conclude that with higher frequency shakers we pick only the fruit, but by reducing the frequency the fruit with the stem. A higher frequency demands a higher thrust $P = M \cdot \omega$, but there are also a number of advantages to decreasing the amplitude, like lessening damage to the tree bark. Excessive increase in frequency leads to detachment of the leaves which, because of its small mass ***m*,** has its own higher frequency than the fruit, according to 39.

CONCLUSION

During agitation of fruit with the shaker the fruit mass does not swing freely but acts harmoniously in a to-and-fro motion of the suspension point at an angle in relation to the tangent, where acceleration also affects the mass.

Tensile force has an effect on fruit detachment parallel to the stem, but tangential force does not directly affect detachment. It is by increasing the angular velocity of oscillation that the indirect normal force is increased, where as is in speed the lowest is in an end position, and the highest is in a vertical position.

Normal force has the greatest value when the fruit passes through the vertical position, where $\cos \omega t = 1$, ($\cos \varphi = 1$ and that is for $\varphi = 0$).

With fruit maturation the connection between the stem and fruit weakens, where the stem dries and rigidity is increased, which leads to more frequent fruit oscillation. Fruit with longer stems are harder to shake off from the fruit than fruit with shorter stems due to centrifugal force.

During analysis of the described oscillation, branch swaying is equal to the radius ($l = r$), where a larger reduction of length leads to an increase in rigidity as well as an increase in its own angular frequency; for example, a four times shorter length has a two times greater frequency. From our analysis we can conclude that in higher frequencies of the shaker only the fruit detach, but with a lesser frequency the fruit along with the stem fall off.

In fruit harvesting with a higher frequency, a higher-powered shaker is required as well as an increase in leaf detachment because of the small mass and its own higher frequency. However, it enables on the other hand a decrease in amplitude, which positively has an effect on the decrease in damage to the tree bark.

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