

The Role of Mathcad in Engineering Education

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This paper describes the solvers of Mathcad that can be used for solving equations and systems of equations in symbols and numerically. This paper is useful for engineering students, teachers and engineers, who have to solve equations and its systems

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Introduction

Everyone wants to solve a math problem the best way. But many of us are mistaken in thinking that for sure they will escape the difficulties if they solve problems using mathematical package (for example, Mathcad or other). Mathematical package is just the best assistant in computing problems but not self-sufficient object. That's why without an understanding of the nature and behavior of each built-in function or operator the full success in the decision can't be achieved. Of course, someone can hammer a screw into the wall not knowing that he needs to use a screwdriver. But can we say, that the work is well done? We don't think so. Was the calculation performed optimally, if it was done by arbitrarily selected means? We think that is unlikely. Many problems in mathematics, physics, chemistry, and other school and university subjects are reduced to solving equations and systems of equations. Therefore it is useful to learn what tools of solving such problems there are in package Mathcad, which is very popular among pupils, students, engineers and scientists. These tools are combined into a group "Solving" among the built-in functions of Mathcad. They use different numerical methods. In Mathcad 15 you can find out the name of the method if you right-click the name of the function. In addition to numerical methods Mathcad implements the analytical method of solving a number of problems. We'll describe these tools using simple school problems about the movement of boats on the river.

1. On symbolic solution of equations by using the keyword "solve" of Mathcad

Let us consider the following physical problem: A motorboat drove down a river the distance $L = 10 \text{ km}$ and then back to the starting point. Full travel time was $t = 105 \text{ min}$. Calculate the velocity of river flow x , if it is known that the speed of the boat in still water is $v = 12 \frac{\text{km}}{\text{h}}$

The travel time down the river is $\frac{L}{v+x}$ and up the river is $\frac{L}{v-x}$. If total the time of moving a boat is t , then the equation for determine the velocity x of a river flow is

$$\frac{L}{v+x} + \frac{L}{v-x} = t \quad (1)$$

Figure 1 shows the symbolic solution of this problem on the worksheet of Mathcad 15

$$\frac{L}{v+x} + \frac{L}{v-x} = t \text{ solve, } x \rightarrow \left[\begin{array}{c} \frac{\sqrt{-t \cdot v \cdot (2 \cdot L - t \cdot v)}}{t} \\ \frac{\sqrt{-t \cdot v \cdot (2 \cdot L - t \cdot v)}}{t} \end{array} \right]$$

Fig. 1. Symbolic solution of the problem on the worksheet of Mathcad

As the velocity of river flow must be $x > 0$ then from equation in Fig. 1 one can get that the required velocity is $x = \frac{\sqrt{tv(tv-2L)}}{t}$. Fig. 2 shows the numeric result according to symbolic solution

$$h := 60 \cdot \text{min} \quad L := 10 \cdot \text{km} \quad t := 105 \cdot \text{min} \quad v := 12 \cdot \frac{\text{km}}{\text{h}}$$

$$x := \frac{\sqrt{t \cdot v \cdot (t \cdot v - 2 \cdot L)}}{t} \quad x = 2.619 \cdot \frac{\text{km}}{\text{h}}$$

Fig. 2. Numeric result according to symbolic solution

The question is why we need to use the numerical mathematics in Mathcad if the problem can be solved simply and beautifully with the help of symbolic mathematics? Clearly, the symbolic solution is aimed to get all possible solutions in the form of formulas (absolute accuracy!). But with more complicated equations, it is often not possible, as is shown in Fig. 3 and 4.

$$\frac{L}{v+x} + \frac{L}{v-x^2} = t \text{ solve, } x \rightarrow$$

The symbolic result returned is too large to display, but it can be used in subsequent calculations if assigned to a function or variable.

Fig. 3. Attempt to solve more complicated equation

Fig. 4 shows that symbolic solution may not be found out at all

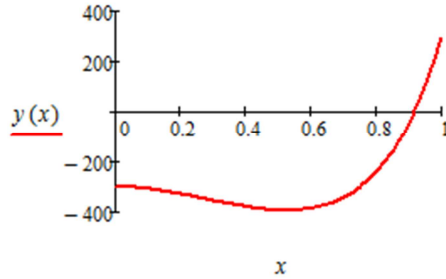
$$\frac{L}{v+x^2} + \frac{L}{v-x^3} = t \text{ solve, } x \rightarrow$$

No solution was found.

Fig. 4. A case, when the symbolic solution is not found

If into the constants of this "non-physical" equation we substitute dimensionless numerical values, at least one real root of this equation can be found out.

Fig. 5 shows the graphical and numeric solutions of equation $y(x) = 0$, where $y(x) = \frac{L}{v+x^2} + \frac{L}{v-x^3} - t$, $L = 10000$, $t = 6300$, $v = \frac{10}{3}$.



$$y(x) = 0 \text{ solve, } x \rightarrow \begin{pmatrix} 0.91753907432064754652 \\ -0.10072745491075931273 - 0.50447307772368065275i \\ -0.10072745491075931273 + 0.50447307772368065275i \\ -0.35804208224956446053 - 1.4320676622180746717i \\ -0.35804208224956446053 + 1.4320676622180746717i \end{pmatrix}$$

Fig. 5. Solution of the equation $y(x) = 0$ graphically and numerically

Fig. 5 shows that the equation $y(x) = 0$ has one real solution in the segment $[0.8, 1.0]$ and 4 complex solutions.

2. On using the function $\text{root}(y(x), x, a, b)$ for solving equations

Fig. 7 shows that specification of the solution by using function $\text{root}(y(x), x, a, b)$ that is coincide with real solution

$$y(x) := \frac{L}{v+x^2} + \frac{L}{v-x^3} - t \quad TOL := 10^{-3}$$

$$x := \text{root}(y(x), x, 0.8, 1.0) = 0.91753907432064030 \quad y(x) = -2.001 \times 10^{-11}$$

Fig. 6. Specification of the solution of equation $y(x) = 0$, when $TOL = 10^{-3}$ by default
The accuracy of the solution depends from the value of variable TOL . By default is $TOL = 10^{-3}$. The value of TOL can be redefined on the worksheet. Fig 7 shows the solution with $TOL = 10^{-15}$

$$y(x) := \frac{L}{v+x^2} + \frac{L}{v-x^3} - t \quad TOL := 10^{-15}$$

$$x := \text{root}(y(x), x, 0.8, 1.0) = 0.91753907432064740 \quad y(x) = 0$$

Fig. 7. Specification of the solution of equation $y(x) = 0$, when $TOL = 10^{-15}$

Fig. 8 shows the graph of the function $y(x) = \frac{\sin(x)}{x}$.

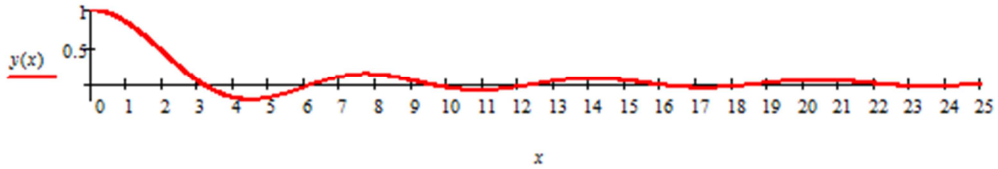


Fig. 8. The graph of the function $y(x)$

Fig. 8 shows that in the segment $[a, b]$, where $a = 2$ and $b = 7$ the equation $y(x) = 0$ has two solutions. Fig. 9 shows that the function $\text{root}(y(x), x, a, b)$ can't determine these solutions, because the function $y(x)$ has the same sign at the end points of this segment

$$\text{root}(y(x), x, 2, 7) = \text{[]}$$

The function values on the end points must be of opposite signs.

Fig. 9. An attempt to solve the equation $y(x) = 0$ in the segment $[2, 7]$

Let us consider the segment $[1, 17]$ the function $y(x)$ has different signs at the endpoints of this segment. Fig. 10 shows, that in this case the function $\text{root}(y(x), x, a, b)$ returns only one solution 9.425.

$$\text{root}(y(x), x, 1, 17) = 9.425$$

Fig. 10. An attempt to solve the equation $y(x) = 0$ in the segment $[1, 17]$

4. On using the function **polyroots(v)**

If in the equation $y(x) = 0$ the function $y(x)$ is a polynomial then it is possible find all solutions by using function “polyroots(v)”, where v is the vector of coefficients of the polynomial. Let us consider again the rational function $(x) = \frac{L}{v+x} + \frac{L}{v-x} - t = \frac{p(x)}{v^2 - x^2}$, where $p(x) = 2Lv - tv^2 + tx^2$ is quadratic polynomial. Equations $y(x) = 0$ and $p(x) = 0$ have identical solutions. Fig. 11 shows the using of function “polyroots(v)”, when $L = 10000$, $t = 6300$, $v = \frac{10}{3}$.

$$w := p(x) \text{ coeffs}, x \rightarrow \begin{pmatrix} 2 \cdot L \cdot v - t \cdot v^2 \\ 0 \\ t \end{pmatrix} \quad \text{polyroots}(w) = \begin{pmatrix} -0.727 \\ 0.727 \end{pmatrix}$$

Fig. 11. Solution the equation $p(x) = 0$ by using function “polyroots(v)”

5. On using solve block “Given-Find”

Let us consider the first problem: From two ports on the straight section $L = 10$ km of the river towards each other simultaneously depart two boats. They meet at distance d from the port 1 dividing the distance L between the ports of the river in the

golden ratio [2] $\frac{d}{L-d} = \frac{L-d}{L}$, where $L - d \neq d$. Find the distance d . The solution is in Fig. 12.

$$\frac{d}{L-d} = \frac{L-d}{L} \text{ solve, } d \rightarrow \left[\begin{array}{c} \frac{L \cdot (\sqrt{5} - 3)}{2} \\ \frac{L \cdot (\sqrt{5} + 3)}{2} \end{array} \right] \left[\begin{array}{c} \frac{L \cdot (\sqrt{5} - 3)}{2} \\ \frac{L \cdot (\sqrt{5} + 3)}{2} \end{array} \right] = \begin{pmatrix} 3.82 \\ 26.18 \end{pmatrix} \cdot km$$

Fig. 12 Finding the distance d

Fig. 12 shows two solutions. Only the first of them (3.82 km) fits us. The second solution (26.18 km) lies outside the considered segment $L = 10 \text{ km}$.

Second problem: Find the speed of the second boat v_2 and the rate of flow of water in the river v when they meet at distance a (found out in the first problem), if the velocity of the first boat is $v_1 = 12 \frac{km}{h}$, the distance between the ports is $L = 10 \text{ km}$ and the movement time is $t = 30 \text{ min}$. This problem models the following simple linear system of equations:

$$t(v_1 - v) = a \quad t(v_2 + v) = L - a \quad (2)$$

where from the first problem $a = \frac{L(3-\sqrt{5})}{2}$. The symbolic solution of this simple system of linear equations on the worksheet of Mathcad by using solve block “Given-Find” is presented in Fig 13

$$\text{Given } t \cdot (v_1 - v) = a \quad t \cdot (v_2 + v) = L - a \quad \text{Find}(v_2, v) \rightarrow \begin{pmatrix} \frac{L - t \cdot v_1}{t} \\ \frac{\sqrt{5} \cdot L - 3 \cdot L + 2 \cdot t \cdot v_1}{2 \cdot t} \end{pmatrix}$$

Fig. 13. Symbolic solution of linear system (2) by using solve block “Given-Find”
From Fig. 13 it follows that the solution of the problem is

$$v_2 = \frac{L - t v_1}{t}, \quad v = \frac{\sqrt{5} L - 3 L + 2 t v_1}{2 t}. \quad (3)$$

Numeric results are presented in Fig. 14

$$v_2 := \frac{L - t \cdot v_1}{t} \quad v := \frac{\sqrt{5} \cdot L - 3 \cdot L + 2 \cdot t \cdot v_1}{2 \cdot t} \quad v_2 = 8 \cdot \frac{km}{h} \quad v = 4.361 \cdot \frac{km}{h}$$

Fig. 14. Computation of numeric result on the worksheet of Mathcad

There is possible also direct numerical solution (Fig. 15) of system (2) by using solve block “Given-Find”. In this case the initial values for unknown parameters v_2 and v are needed.

$$\text{Given } t \cdot (v_1 - v) = a \quad t \cdot (v_2 + v) = L - a \quad \begin{pmatrix} v_2 \\ v \end{pmatrix} := \text{Find}(v_2, v) = \begin{pmatrix} 8 \\ 4.361 \end{pmatrix} \cdot \frac{km}{h}$$

$$t \cdot (v_1 - v) - a = 9.095 \times 10^{-13} m \quad t \cdot (v_2 + v) - L + a = 9.095 \times 10^{-13} m$$

Fig.15. Direct numeric solution of system (2) on the worksheet of Mathcad

In direct numeric solution of an equation or system of equations with the solve block “Given-Find” the accuracy of solution may depend on value of the parameter *CTOL*. By default $CTOL = 10^{-3}$. The value of *CTOL* can be redefined directly on the worksheet. The site <http://communities.ptc.com/videos/1472> shows visually the search of solutions of a system of two nonlinear equations by method of Newton. The site <http://communities.ptc.com/videos/2418> shows how the initial values influence the solution. Details about using the solve-block can be found in [1].

6. On using function *lsolve*(*M*, *v*)

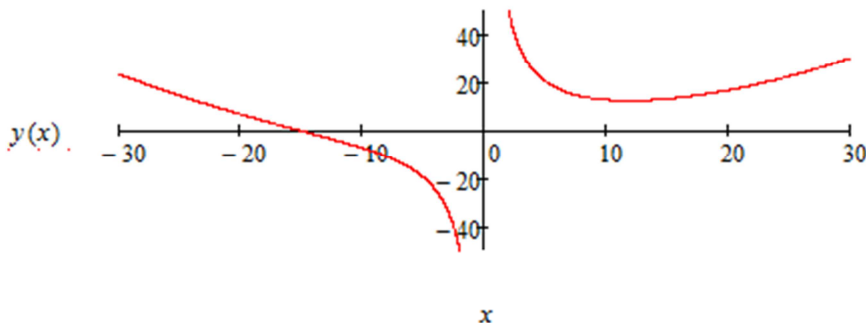
Equations (2) can be written in the matrix form $M \begin{pmatrix} v_2 \\ v \end{pmatrix} = V$, where $M = \begin{pmatrix} -t & 0 \\ t & t \end{pmatrix}$, $V = \begin{pmatrix} a - t v_1 \\ L - a \end{pmatrix}$. Fig. 16 shows the solution the system of linear equations (2) by using function *lsolve*(*M*, *v*) and by using inverse matrix

$$\begin{pmatrix} v_2 \\ v \end{pmatrix} := \text{lsolve}(M, V) = \begin{pmatrix} 8 \\ 4.361 \end{pmatrix} \cdot \frac{km}{h} \quad M^{-1} \cdot V = \begin{pmatrix} 8 \\ 4.361 \end{pmatrix} \cdot \frac{km}{h}$$

Fig. 16. Solution of the linear algebraic system on the worksheet of Mathcad

7. On using function “Minerr (x, y,...)”

If the solve block “Given-Find” can't solve the system of equations, it returns an error message. The solve block “Given-Minerr” returns the solution or the last approximation with minimal quadratic error for the solution. Let us consider function $y(x) = \frac{100}{x} + \frac{3}{100} x^2$. In Fig 17 the solve block “Given-Minerr” found the minimum value of the function *y*(*x*).



Initial value $x := 20$ Given $y(x) = 0$ $x := \text{Minerr}(x) = 11.856$ $y(x) = 12.651$

Fig. 17. Determination of minimum value of function *y*(*x*)

In Fig. 18 the solve block “Given-Minerr” found out the solution of equation $y(x) = 0$.

Initial value $x := -20$ Given $y(x) = 0$ $x := Minerr(x) = -14.938$ $y(x) = 0$

Fig. 18. Determination the solution of equation $y(x) = 0$

Conclusion

If one is solving a standard and not very significant problem, then you need not waste time on the selection of the correct method. But if you are a serious researcher, you'll make the best choice. The best outcome is to get common symbolic solution. It is important to understand why very often we can't solve the equation symbolically and how we can and should help the computer to solve the equation, whether symbolically or numerically. In order to successfully solve their school assignments or professional tasks with the help of computer pupils, students, engineers or scientists need to learn the solvers of Mathcad as well as the properties of numerical, graphical and analytical problem-solving techniques.

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