# MATHEMATICAL DESCRIPTION FOR THE MOULDBOARD PLOUGH SURFACE AND THE SOIL SLICE PATH DURING THE PLOUGHING OPERATION

**V. Craciun, M. Stancu, D. Leon**

**Abstract:** *In the paper the authors presents a new mathematical description of the cylindroidal moldboard plough surface of the plough bottoms. This parametric description, together with an adequate model of interaction between the soil particles and moldboard surface, permit to determine the differential equations for the soil slice path on the plough working surface. Using numerical methods for solving the equations and comparing the theoretic with experimental path one can make the prediction of soil reaction forces on a mouldboard working surface and optimize the tools.*
***Key words:*** *plough, soil- tool interactions, soil path, mouldboard, mathematic description.*

## INTRODUCTION

The mouldboard ploughs are the main agricultural implements for tillage and one of the biggest consumers of energy in agriculture. The mouldboards are complex spatial surfaces and their geometry, varying in function of the type of the mouldboard and purpose, determine the quality of the working process and the energy consumption. In order to predict the soil resistance in the time of plowing are necessary to know the mathematical description of the mouldboards and an accurate evaluation of the soil-machine interactons.

Today the mathematical description of the working surface of the plough bottoms which consists from the share and mouldboard is not entirely solved. Besides, are not solved the theoretical problems of the movement of the soil slices on the working surfaces of the mouldboard plough bottoms. In the specialized literature there are many attempts to describe the moldboard plough shape [Richey, S at al.; 1989, Suministrado, D.C. at all 1990;Ros, V. at al., 1995 etc.]. These studies inadequately represent the surfaces for the design purposes and are difficult to be implemented for the CAD- CAM methods. Generally, ***"the development of adequate means to describe the shape of tillage tools is still a major area of concern in the design of tillage tools"*** [Ros, V. at al., 1995, Jie Shen & Radley Lal Kushwaha, 1998]. Considering a mouldboard of cylindroidal type the authors developed a mathemathical description (parametrical) for the mouldboard shape. Assuming a mathematical algoritm concerning soil-mouldboard interactions the authors developped the equation which describe the soil path on the mouldboard working surface.

## NOTATIONS

$\Sigma$ **-** cylindroidal surface;
$\Delta$ **-** generatrix line;
$\Gamma$ **-** directrix curve;
$\gamma(z)$**-** the angle between generatrix line and the furrow wall at the height $z$;
$V_m$ – forward speed of the plough;
oxyz- coordinate system fixed on the soil;
WXYZ –coordinate system, named transportation system, fixed on the moving mouldboard;
$O_1x_1y_1z_1$ –Cartesian coordonate system, perpendicular on the share edge, containing diretrix curve $\Gamma$;
***"u"***- scalar parameter;
***"v"*** – scalar parameter on the generatrix line $\Delta$;
$\bar{a}$ **-** position vector;

$\bar{b}$ - position vector on the generatrix line ***Δ;***
$\overline{WA} = \bar{R}$ - position vector;
A,B,C,D - the coefficients for the polynomial of the directrix curve, ***Γ;***
m, n, p, q – the coefficients of the function ***γ(z);***
$\gamma_0$ - the angle between the share edge and the furrow- wall (see fig.1)
$\theta$ - the angle between the share plane and horizontal plane XOY;
***mg-*** soil weight, acting in vertical direction;
μ - the coefficient of soil- methal friction;
$F_n$ – the mouldboard reaction force;
$\bar{\upsilon}$ - normal vector to the mouldboard surface, on the soil path, fig.2;
$\bar{\tau}$ - unit vector, tangent to the to the mouldboard surface, on the soil path, fig.2;
$\overline{F}_c$ - cohesion force;
$\overline{F}_{ad}$ -soil- methal adhesion force;
$\overline{F}_a$ - resultant of the active forces;
$\overline{F}_p$ - resultant of the passive forces;
-$a_t$, transport acceleration, $a_t = 0$;
-$a_{cor}$ , Corriolis acceleration;
$\bar{a}_r$ - relatrive acceleration of the movementof the soil particule;
$\bar{i}, \bar{j}, \bar{k}$ - unit vectors, fig.1.

## A NEW MATHEMATICAL DESCRIPTION FOR THE CYLINDROIDAL MOLDBOARD PLOUGH SURFACE

In this paper the authors have developed a parametric description for the cylindrodal mouldboards. A cylindroidal surface ***(Σ)***, fig.1, is generated by a generatrix line ***(Δ)***, which is moving on a directrix curve ***(Γ)***. While moving, the generatrix line ***(Δ)*** is always parallel to the horizontal plane XWY and it determines a variable angle ***γ(z)*** with the YWZ vertical- longitudinal plane. The mouldboard is a part from this cylindroidal surface with the boundaries determined by the strict rules (Bernacki at al., 1972, Caproiu at al. 1982, Craciun & Leon, 1998). Refer to the fig 1 we must give the following conditions:

-the orthogonal coordinate system of axis ***Oxyz*** is fixed on the soil;

-the orthogonal coordinate system ***WXYZ*** (named transportation system) is fixed with the moving mouldboard. Its axis are parallel with the axis of the Oxyz system and is moving with the speed $V_m$ , the speed of the ploughing aggregate, in the sense ***Oy ≡ WY***.

**-"u"** is a scalar parameter, in our case is identical with the height **"z"**;

$$u \equiv z \equiv z_1 \equiv Z \qquad (1)$$

**-"v"** is a scalar parameter; in this case is a Cartesian coordinate on the generatrix line ***(Δ)***;

-***"Γ"***, the directrx curve, is defined in the coordinate system $O_1x_1y_1z_1$ , perpendicular on the share cutting edge. The $O_1x_1y_1z_1$ plane is located at the distance $L_1$ from the share point. The directrix curve ***"Γ"*** is reasonable defined by a third degree function [Craciun & Leon, 1998]:

$$x_1 = Au^3 + Bu^2 + Cu + D = f_1(u) \qquad (2)$$

**-"$\gamma_i$"** is the angle between the directrix line ***"Δ"*** at the height ***($z_i$)*** with the plane ***YWZ*** (furrow wall). A very good relationship for the variation of the ***γ (z)*** is given by a third degree function too:

$$\gamma(z) = mz^3 + nz^2 + pz + q = f_2(u) \qquad (3)$$

as recommended by Craciun and Leon [1998].

For $z = u = 0$, $\gamma(z) = \gamma_0$, the angle between the share cutting edge and the furrow wall (direction of travel);

-$\theta$, is the angle between the share plane and the horizontal plane ***XWY***, fig.1.

A certain point M on the **directrix** curve *"Γ"* have a position vector

$\overline{WM} = \bar{a}$ , where, $\bar{a} = \bar{a}(u)$ (4)

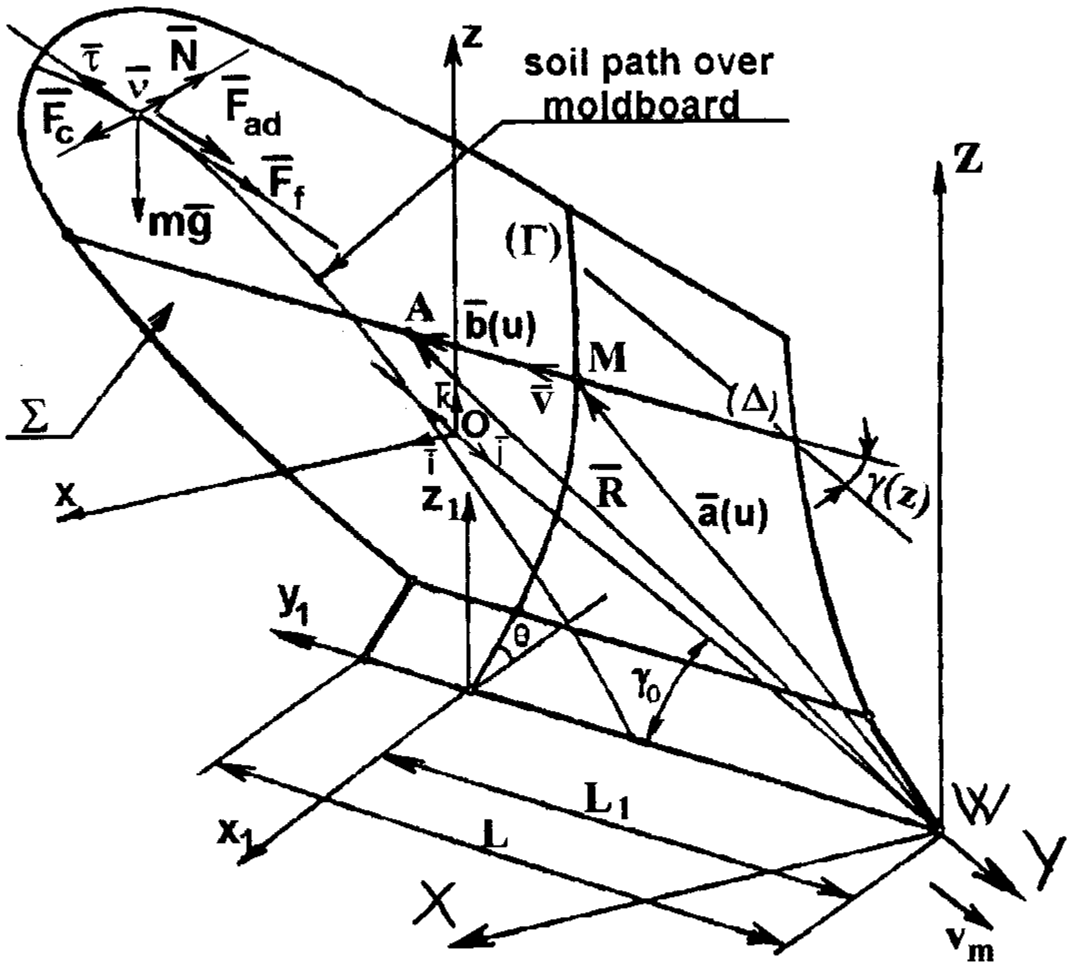


**Fig.1. Perspective of a moldboard surface; forces acting on soil particle and the soil path**

A generic point ***A***, of the moldboard, located on the generatrix line "Δ" is positioned by the scalar parameter ***"v"*** in function by the coordinates. The unit vector $\bar{b}$ for the point ***A*** is a function of the parameter ***"u"***, $\bar{b} = \bar{b}(u)$ , hence;

$$\overline{MA} = \bar{b}(u) \cdot \mathrm{v} \qquad (5)$$

Consequently, the position vector $\overline{WA} = \bar{R}$ will have the expression:

$$\bar{R} = \bar{a}(u) + \bar{b}(u) \cdot \mathbf{v} \qquad (6)$$

The expression (6) is the parametric form of the vectorial equation of the cylindroidal surface in the ***WXYZ*** coordinate system. To obtain the equivalent scalar equations for the equation (6) one must find the equations for the functions ;

$$X=X(u, v), Y= Y(u, v), Z= Z(u, v) \qquad (7)$$

Taking in to considerations the notations given in the fig.1, the forces acting on the soil particule along the soil path on the mouldboard sourface, as is shown in fig.2 , and through an adequate mathematical treatment, the equations for the cylindroidal working surface of the mouldboard plough bottom are:

$$\begin{cases} X(u,v) = f_1(u)\cdot\cos\theta_0 - v\cdot\sin f_2(u) \\ Y(u,v) = f_1(u)\cdot\sin\theta_0 + v\cdot\cos f_2(u) \\ Z(u,v) = u \end{cases} \quad (8)$$

## DYNAMICS OF THE MOVEMENT OF THE SOIL PARTICLES ON THE MOLDBOARD SURFACE

There are some researches concerning mathematical models for determining the trajectories of the particles over moldboard surface (Suministrado et al., 1990), but these researches "inadquately represent the surfaces for the design purposes" (Ros etal.,1995).

A mathematical description of the tillage process can be accomplished only when all the elements of the process are expressed in a quantitative sense (Gill et al, 1967, Ros et al.,1995). Having mathematical description of the moldboard, together with the soil- tool interaction given in the fig.1 and with initial soil conditions and soil movement, applying the methods of mechanics, the authors have determined a system of differential equations that describe the trajectories of the soil on moldboard plough surface.

To simplify the problem the authors consider that:

-the soil mass is formed only from distinct particles; thus we can neglect the proper rotations and the dynamic effects of these rotations;

-the soil particles are moving only in to the vertical- transverse plane ***XWZ*** [Bernacki, 1972]. Fig.2 shows the forces that act on a soil particle as the moldboard advances:

-***mg***, soil weight, acting in the vertical direction, ***(N);***

-the moldboard reaction force $\boldsymbol{F_n}$, acting in the direction of normal vector $\bar{\upsilon}$,on the moldboard surface, ***(N);***

-the soil-metal friction force, $\bar{F}_f, (|\bar{F}_f| = \mu|F_n|)$, oriented in the direction of the unit vector $\bar{\tau}$ of the tangent, ***(N);***

-$\bar{F}_c$, cohesion force, with the direction of the normal vector $\bar{\upsilon}$ and sense opposite to $F_n$, ***(N);***

$\bar{F}_{ad}$, soil-metal adhesion force, acts in the same direction and sense with friction force $\boldsymbol{F_f}$ .

To determine the equations of the movement of the soil particle on the moldboard we will use the equations (7).

The relative movement of the particle on the moldboard surface take place being governed by the general equation known from the mechanics:

$$m\cdot\bar{a}_r = \bar{F}_a + \bar{F}_f - m\cdot\bar{a}_t - m\cdot\bar{a}_{cor} \quad (9)$$

where, $\bar{a}_r$, is the relative acceleration of the movement, which has the expression:

$$\bar{a}_r = \ddot{X}\cdot\bar{i} + \ddot{Y}\cdot\bar{j} + \ddot{Z}\cdot\bar{k} \quad (10)$$

-$\bar{F}_a = m\cdot\bar{g}$, is the resultant of the active forces;

-$\bar{F}_p$, is the resultant of the passive forces;

$$\bar{F}_p = \bar{F}_c + \bar{F}_n + \bar{F}_f + \bar{F}_{adc} \quad (11)$$

-$a_t$, transport acceleration, $a_t = 0$;

-$a_{cor}$ , Corriolis acceleration, in this case, $a_{cor} = 0$.

With the clarifications given above, the equation (8) became:

$$m(\ddot{X}\bar{i} + \ddot{Y}\bar{j} + \ddot{Z}\bar{k}) = -mg\cdot\bar{k} + (F_n - F_c)\cdot\bar{\upsilon} - (F_f + F_{ad})\bar{\tau} \quad (12)$$

The unit vectors $\bar{\upsilon}$ and $\bar{\tau}$ will be expressed as functions of the unit vectors $\bar{i}, \bar{j}, \bar{k}$. Using proper mathematical description for the normal and tangent vectors to a given surface (here, the moldboard) and taking into consideration the assumption that the particle of soil are moving only in ZOX plane, y = const., and knowing that:

$$y = Y_w + Y, \quad \text{and } Y_w = V_m.t \tag{13}$$

where $V_m$ is the speed of the aggregate, results:

$$\dot{Y} = -V_m, and \quad \ddot{Y} = 0. \tag{14}$$

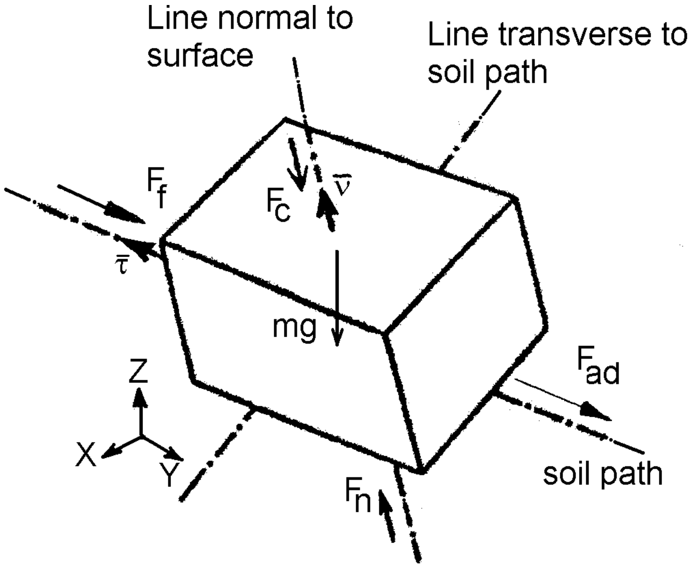


Fig.2. An elemental soil block on the mouldboard surface and the acting forces

With these clarifications, the differential equations that describe the soil path of the particle on the moldboard are (15):

$$\begin{cases} 0 = m\ddot{X} + \frac{(\mu F_n + F_{ad})}{|V|} \cdot \left[\dot{X} - \frac{V_m}{tg\ f_2(u)}\right] \\ 0 = m\ddot{Z} + mg - \frac{(\mu F_n + F_{ad})}{|V|} \cdot \\ \cdot \left\{\dot{Z} + \frac{V_m}{\sin f_2(u)}\left[v f_2'(u) - f_1'(u)\cos(f_2(u) - \gamma_0)\right]\right\} \end{cases} \tag{15}$$

where:

$$V = \left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right)^{1/2} = \left(\dot{X}^2 + V_m^2 + \dot{Z}^2\right)^{1/2} \tag{16}$$

**u** and **v** being the scalar parameters which define the ruled surface of the moldboard. The system (15) is non-linear and includes the complex geometry of the moldboard ($\gamma_0, \gamma, f_1(u), f_2(u), f_1'(u), f_2'(u), u, v$), the soil conditions ($\mu$, Fc, $F_{ad}$) and the working speed of the plough, $V_m$.

**CONCLUSIONS AND FUTURE WORK**

As a results of the investigations made by the authors the folowing conclutions can be made:

1. Considering a cylindroidal mouldboard surface, have been determined the parametric equations of the mouldboard (7) surface in an orthogonal system of axis, fig. 1.

2. Having mathematical description of the moldboard plough surface, together with an adequate soil-tool interactions model, the authors determined the differential equations for the soil particle path on the plough bottom working sourface. Using

numerical methods for integration and computer adapted programs one can determine the trajectory lines of the soil particle on the mouldboard surface.

3. Farther investigation are still necessary as regards to improve the model of soil- tool interaction and predict the draft force and finally, for the optimization the tools surface.

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